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A NEO-CAMBRIDGE MODEL OF INCOME DISTRIBUTION AND UNEMPLOYMENT

by

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1. Introduction

The unemployment ratio and the share of wages have been of central concern to most economists who would be classed as Post Keynesians. This is especially true of those Cambridge (UK) economists who have been most instrumental in developing an alternative to conventional neo-classical macrodynamics. In this paper, by putting together some observations and assumptions associated with four Cambridge macroeconomists--Goodwin, Kahn, Kaldor and Joan Robinson--we generate a disequilibrium dynamic model of the interaction between the share of wages and the (un-)employment ratio.

The essential feature of the behavioural mechanism of the model, to be truly neo-Cambridge, is that technical progress and its interaction with capitalistic behaviour about installation of new capacity (investment) and utilization of existing capacity together with the attempt by the working class to alter the functional income distribution in their favour should generate a dynamical system in the two variables of interest--the (un-)employment ratio and the share of wages.

Thus, by making assumptions about

i) Money-wage dynamics (as a bargaining mechanism)
ii) Pricing
iii) Technical progress
iv) Investment
v) Capital-output ratio (capacity utilization)

associated with Goodwin (1967, 1970, 1972), Kahn (1972, essays 7 and 9), Kaldor (1960, 1961, 1962) and Robinson (1956), it is shown that a very general, <u>non-linear</u> dynamical system in the share of wages and the employment ratio can be generated. Some properties, e.g., uniqueness, existence and stability (both asymptotic and structural), of the equilibrium for the system, are discussed where it is shown that the model is essentially policy oriented though it is a 'pure capitalist' economy that is being modelled in the sense of not including a specific role for government.

2. The Model

We assume a closed economy producing one industrial good (or product of manufacturing industry) with no explicit role for government; the economy is basically noncompetitive.

Notation:

$$v = \frac{L}{N} \tag{1}$$

$$\frac{\dot{v}}{v} = \frac{\dot{L}}{L} - \frac{\dot{N}}{N} \tag{2}$$

where: L = level of employment

N = level of population (or labour supply)

v = employment ratio

$$u = \frac{ml}{p} \tag{3}$$

$$\frac{\dot{u}}{u} = \frac{\dot{m}}{m} - \frac{\dot{p}}{p} - \left(\frac{\dot{Y}}{Y} - \frac{\dot{L}}{L}\right) \tag{4}$$

where: u = share of wages

m = money wage rate

p = price level

Y = level of output

(i) <u>Money-wage dynamics</u>:

<u>Assumption 1</u>:

$$\frac{\dot{m}}{m} = f(v) + g_1\left(\frac{\dot{p}}{p}\right) \tag{5}$$

where $f \; \varepsilon \; C'$ and $g \; \varepsilon \; C'$

and $f' > 0$, $g'_1 > 0$, $m \geqq \bar{m} >> 0$

Thus, following Goodwin (1967, 1970, 1972), Kahn (1972, ch. 7), Kaldor (1959) and Robinson (1956, ch. 20), we relate growth (proportional) in money wages to some function of the employment ratio (proxy for level or rate of unemployment) and growth in prices. This is, of course, Phillips-Curve wisdom. That there is empirical justification for such a relation is discussed in Santomero-Seater (1978). The Cambridge rationale for such a relation would be in terms of an underlying bargaining mechanism.

(ii) <u>Pricing</u>:

<u>Assumption 2</u>:

$$\frac{\dot{p}}{p} = \lambda(\log m - \log p + \log \pi + \log Y + \log L) \tag{6}$$

i.e., $$\frac{\dot{p}}{p} = \lambda(\log \pi + \log u) \tag{7}$$

where $\pi > 1$ (the mark-up factor)

and $\lambda > 0$ (the adjustment coefficient)

More generally, we may write (7) as:

$$\frac{\dot{p}}{p} = h(u; \lambda, \pi) \tag{8}$$

where $h'_u > 0$ and $h \; \varepsilon \; C'$

This is simply a mark-up on unit costs assumption (cf.

Kaldor (1970), p. 3, Kaldor (1959), pp. 216-220, Robinson (1956), p. 179)--with the additional proviso that observed prices adjust to equilibrium prices in an exponentially distributed lag form (cf. also Rowthorn (1977), p. 218 and fn. for a lucid verbal explanation of such adjustments).

(iii) Technical Progress Function:

For the great classical economists and Marx, unemployment was closely linked to, and, indeed even solely generated by--in some cases--the interaction between technical progress and accumulation (i.e., investment). The Cambridge economists have quite explicitly carried on this distinguished tradition. The neo-classical detour via a production function, in dynamical systems, is not only unnecessary but also insufficient (cf. Hildenbrand (1981), in particular p. 1108).

Assumption 3 (The Technical Progress Function):

$$\frac{\dot{Y}}{Y} - \frac{\dot{L}}{L} = \zeta\left(\frac{\dot{K}}{K} - \frac{\dot{L}}{L}\right) \tag{9}$$

where K: Capacity (Capital)

and $\zeta \in C'$

This relation is, of course, the famous Kaldorian technical progress function where growth in output per man is related to investment per capita (cf. Kaldor (1961), p. 265, Kaldor (1961), pp. 207-208, and Kaldor-Mirrlees). It must be pointed out that the Kaldorian non-vintage, non-linear technical progress function can be derived with only the effect of increasing capital per man playing a fundamental role (cf. Kennedy (1961)).

This, for our purposes, strengthens the justification of using such a function because, as observed in the opening lines of this sub-section, we would like the fluctuations in the employment ratio (the 'reserve army of labour') to depend primarily on the interaction between growth in productivity (technical progress) and accumulation (investment).

(iv) The Investment Function:

Given the Kaldorian idea that:

> . . . the rate of investment in period 1, as a proportion of the income of that period, equals the rate of growth of income over the previous period multiplied by the capital-output ratio of the current period, plus a term depending on the change in the rate of profit over the previous period
>
> (Kaldor (1960), pp. 277-278)

and modifying it appropriately to make it conformable to our continuous-time version as distinct from Kaldor's discrete-time version, we get:

Assumption 4 (The Investment Function):

$$\frac{I}{Y} = \frac{\dot{K}}{Y} = K\,\frac{\dot{Y}}{Y} + g\left((1 - u)\,\frac{Y}{K}\right) \tag{10}$$

i.e.,

$$\frac{\dot{K}}{K} = \frac{\dot{Y}}{Y} + g\left((1 - u)\,\frac{1}{K}\right)\frac{1}{K} \tag{11}$$

where: $I = \dot{K}$

and K: $\frac{K}{Y}$: capital-output ratio

and $g \in C'$

(v) Capital-Output Ratio (or Capacity Utilization):

The simplest possible assumption to account for short-run variations in the degree of utilization of capacity and labour-hoarding, and at the same time leading to "steady capital-output ratios over long periods" (cf. Kaldor, 1961, p. 178) would be to posit a relationship between the observed capital-output ratio, the desired capital-output ratio and the employment ratio (cf. Desai, 1973)

Assumption 5:

$$k = \frac{K}{Y} = f_1(v) \tag{12}$$

where $f'_1 < 0$ and $f_1 \in C'$

where we have subsumed the 'desired capital-output ratio' in the functional form relating 'observed capital-output ratio' with the employment ratio.

As an example of the way in which such a relationship, in any particular form, takes account of less than full utilization of capacity, we may consider the following function suggested by Desai (1973):

$$k = k^* v^{-\mu}$$

where $0 < \mu \neq 1$ and k^*: desired capital-output ratio

Then, whenever employment is less than full, the amount of non-utilized capacity is positive and given by:

$$(k^* v^{-\mu} - k^*) Y$$

From (1) ~ (12) we can derive a 'reduced form' dynamical system in the phase-plane for u and v. From (12) we get:

$$\frac{\dot{K}}{K} - \frac{\dot{Y}}{Y} = \frac{f'_1(v)}{f_1(v)} \cdot \dot{v} \tag{13}$$

substituting (12) in (11) we get

$$\frac{\dot{K}}{K} - \frac{\dot{Y}}{Y} = g\left((1 - u) \frac{1}{f_1(v)}\right) \frac{1}{f_1(v)} \tag{14}$$

From (13) and (14) we get:

$$\frac{f'_1(v)}{f_1(v)} \cdot \dot{v} = g\left((1 - u) \frac{1}{f_1(v)}\right) \frac{1}{f_1(v)} \tag{15}$$

Dividing both sides by v and rearranging, we get:

$$\frac{\dot{v}}{v} = \frac{g\left((1 - u) \frac{1}{f_1(v)}\right)}{v \cdot f'_1(v)} \tag{16}$$

Put:

$$G(v,u) \equiv \frac{g\left((1 - u) \frac{1}{f_1(v)}\right)}{v \cdot f'_1(v)} \tag{17}$$

Then:

$$\frac{\dot{v}}{v} = G(v,u) \tag{18}$$

By substituting (14) in (9) we get:

$$\frac{\dot{Y}}{Y} - \frac{\dot{L}}{L} = \varrho\left(\frac{\dot{Y}}{Y} - \frac{\dot{L}}{L} + g\left((1 - u) \frac{1}{f_1(v)}\right) \frac{1}{f_1(v)}\right) \tag{19}$$

We now make the following assumption (which could, for example, be dispensed with, if we made explicit endogenous assumptions about labour supply--as a function of real wages, for. eg.--or if we assumed that population growth was a given exogenous factor):

<u>Assumption 6</u> (Additive Separable Technical Progress Function):

The technical progress function can be written as an additive function of growth in productivity and the profits term of the investment equation, the latter multiplied by the reciprocal of the capital-output ratio.

Thus:

$$\frac{\dot{Y}}{Y} - \frac{\dot{L}}{L} = \varrho_1\left(\frac{\dot{Y}}{Y} - \frac{\dot{L}}{L}\right) + \varrho_2\left(g\left((1 - u) \frac{1}{f_1(v)}\right) \frac{1}{f_1(v)}\right) \tag{20}$$

Put $$(\frac{\dot{Y}}{Y} - \frac{\dot{L}}{L}) - \mathcal{b}_1(\frac{\dot{Y}}{Y} - \frac{\dot{L}}{L}) \equiv B(\frac{\dot{Y}}{Y} - \frac{\dot{L}}{L}) \qquad (21)$$

Then:

$$B(\frac{\dot{Y}}{Y} - \frac{\dot{L}}{L}) = \mathcal{b}_2\left(g\left((1 - u)\frac{1}{f_1(v)}\right)\frac{1}{f_1(v)}\right) \qquad (22)$$

From (22) we get:

$$\frac{\dot{Y}}{Y} - \frac{\dot{L}}{L} = B^{-1}\left\{\mathcal{b}_2\left(g\left((1 - u)\frac{1}{f_1(v)}\right)\frac{1}{f_1(v)}\right)\right\} \qquad (23)$$

Put $$B^{-1}\left\{\mathcal{b}_2\left(g\left((1 - u)\frac{1}{f_1(v)}\right)\frac{1}{f_1(v)}\right)\right\} \equiv D(v,u) \qquad (24)$$

Now, substituting (5), (8) and (24) in (4), we get:

$$\begin{aligned}\frac{\dot{u}}{u} &= f(v) + g_1(\frac{\dot{p}}{p}) - \frac{\dot{p}}{p} - D(v,u) \\ &= f(v) + g_1\left(h(u;\lambda,\pi)\right) - h(u;\lambda,\pi) - D(v,u) \qquad (25)\end{aligned}$$

Put $$H(v,u) \equiv f(v) + g_1\left(h(u;\lambda,\pi)\right) - h(u;\lambda,\pi) - D(v,u) \qquad (26)$$

where we have suppressed, in H, the explicit indication of its dependence on the parameters $\{\lambda, \pi\}$.

Then, $$\frac{\dot{u}}{u} = H(v,u) \qquad (27)$$

Thus, the dynamical system, for the employment ratio and the share of wages, will be given by (18) and (27). A striking feature in the derivation of (18) and (27) is that no explicit assumption whatsoever has been made about growth in population. The inherent dynamics of the model automatically generates the 'industrial reserve army' (quite in accordance with Marx's views).

3. The Working of the Model

Obviously, to get meaningful results about existence, uniqueness and stability of interesting equilibrium situations we must make more specific and typically 'Cambridge assumptions'--so far as it is possible--about the functional relationships. This we proceed to do now.

<u>Assumption 7</u> (A 'Keynesian' Assumption):

$$g'_{,u} = \frac{\partial g}{\partial u} > 0$$

This means that the increase in the share of wages, through demand effects, will stimulate investment. The reverse assumption would be a 'classical' assumption (in Keynes' sense of the term) in that any increase in the share of wages, by depressing the current (and expected) profit rates, acts as a deterrent in the investment equation (cf. also Kalecki (1971), ch. 14 and Steindl (1952), p. 237). Using assumption 7 in conjunction with assumptions on (12), (17) and (18) we get:

$$\frac{\partial(\frac{\dot{v}}{v})}{\partial u} = \frac{\partial G}{\partial u} > 0 \tag{28}$$

Using (17) a direct evaluation of changes in the proportional growth rate of the employment ratio for infinitesimal changes in the ratio gives:

$$\frac{\partial G}{\partial v} = \frac{\dfrac{v\frac{\partial g,}{\partial v}}{f,\ (v)^2} - \left\{g,\left((1-u)\frac{1}{f,(v)}\right)\left[f'_,(v) - vf''_,(v)\right]\right\}}{v\ f'_,\ (v)^{2}} \tag{29}$$

The denominator is unambiguously positive. Since, by assumption, $f'_, (v) < 0$ and $f''_, (v) > 0$ and $v > 0$, the second term in the numerator (i.e., the terms enlosed with the curly brackets) will be unambiguously negative and hence:

$$-\left\{g,\left((1-u)\right)\left[\frac{1}{f,(v)}\ f'_,(v) - v\ f''_,(v)\right]\right\} > 0 \tag{30}$$

In the first term in the numerator, we have a positive denominator and in the numerator we have $v > 0$. Considering now $\frac{\partial g}{\partial v}$, in conjunction with the reasoning behind assumption 7 we would naturally assume this to be positive. Intuitively speaking, this is another 'Keynesian' or Cambridge' assumption--the investment effects of increases in employment are further increases in investments through the demand effects of the employment multiplier:

> . . . a high demand for labour is associated with a high rate of investment . . .
>
> (Kaldor (1970), p. 5)

We assume, therefore:

<u>Assumption</u> <u>8</u>:

$$\frac{\partial G}{\partial v} < 0 \qquad (31)$$

Now, from (25) and (26) we have:

$$\frac{\partial (\frac{\dot{u}}{u})}{\partial u} = \frac{\partial H}{\partial u} = \frac{\partial g}{\partial h} \cdot \frac{\partial h}{\partial u} - \frac{\partial h}{\partial u} - \frac{\partial D}{\partial u} \qquad (32)$$

Clearly, $\frac{\partial h}{\partial u} > 0$ (cf. (8) and assumptions in conjunction with it). Also $\frac{\partial g}{\partial h}$ could be assumed to be positive.

Since we have assumed that investment is stimulated by increases in the share of labour (at least in a non-full-employment situation - assumption 7), quite naturally it follows that productivity would also be stimulated by increases in the share of labour ('high-wage, high-productive' economy!), i.e. $\frac{\partial D}{\partial u} > 0$.[1] One way of putting these assumptions to-

1. We referred to Kalecki's famous article on 'Class Struggle and Distribution of National Income' below assumption 7. It is important to bear in mind the arguments given in making assumption 7 operational. As it is, the assumption seems counter-intuitive, in particular in a one-good

gether would be to reason as follows: if the proportional growth in money wage rates due to indexation clauses with respect to inflation (price rises) less growth in productivity due to changes in the share of wages is dominated by inflation, then the proportional growth in the share with respect

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economy, closed for foreign trade and without an explicit role for government. The arguments behind assumption 7 become clear in a two- or multi-sector economy, as for example, in Kalecki's typically Marxian 3-sector economy or Joan Robinson's 2-sector models. Thus, as Kalecki argued:

> Until fairly recently it was generally accepted that if wages are raised profits decline PRO TANTO. Even though in the analysis of other phenomena Say's law was not adhered to, at least not strictly, in this case the preservation of purchasing power was not put to doubt. And the analysis of increase or reduction in wage rates dealt with the physical consequences of this absolute shift from profits to wages or VICE VERSA. In the case of the rise in wage rates, the reconstruction of capital equipment in line with the higher spending on wage goods and lower outlays on investment and capitalists' consumption was emphasized; as well as the tendency to higher unemployment as a result of substitution of capital for labour that has become more expensive.
>
> . . .
>
> . . . the argument based on Say's law would . . . prove fallacious--<u>at least with regard to the short period considered</u>.
>
> The last qualification is essential. For it may be argued that the decline in the volume of investment and capitalists' consumption as a result of the wage rise although not immediate would still come about with delay, say, in the next short period. <u>And this would be true if capitalists at least DECIDED to cut their investment and consumption immediately after having agreed to raise wages</u>. But <u>even</u> this is unlikely . . .
>
> (Kalecki (1971), pp. 156-158)
> (My emphasis)

Precisely the same reasoning is followed by Steindl (op. cit.):

> . . . the increase of wages could never reduce profits as long as investment and capitalist consumption remain high . . .
>
> (Steindl (op. cit.), p. 237)

to changes in the share is negative. <u>This brings the redistributive role of inflation very clearly to the surface</u>. When, therefore, inflation has this strong redistributive role we can assume:

<u>Assumption 9</u>:

$$\frac{\partial H}{\partial u} < 0 \tag{33}$$

In a sense, this assumption compensates for the 'counter-intuitive' assumption 7. These two assumptions (i.e., 7 and 9) are, perhaps, reflected in Marx:

> A rise in the price of labour, as a consequence of accumulation of capital, only means, in fact, that the length and weight of the golden chain the wage-worker has already forged for himself, allow of a relaxation of the tension of it . . .
>
> . . .
>
> The rise of wages therefore is confined within limits that not only leave intact the foundations of the capitalistic system, but also secure its reproduction on a progressive scale.
>
> (Marx (1965), pp. 618-620)

The conflict implied in having assumptions 7 and 8 working together within the same model underlies <u>the common strands in a Marx inspired Kaleckian model and a Keynes inspired Cambridge model</u> (Nuti (1977)) <u>on conclusions about functional income distribution and employment</u>.

Now, from (25) we have:

$$\frac{\partial(\frac{\dot{u}}{u})}{\partial v} = \frac{\partial H}{\partial v} = f'(v) - D_v'(v,u) \tag{34}$$

where $$D_v'(v,u) = \frac{\partial D}{\partial v} \tag{35}$$

Clearly, in the neighbourhood of full employment, the first term on the r.h.s. of (34) would be quite large (from the 'Phillips curve wisdom'). The second term on the r.h.s. of (34) is not easy to interpret. A possible line of argument would be as follows: In the early stages of the up-turn there would be large gains in labour productivity for small increases in the employment ratio due to labour-hoarding. As full employment is approached the gains in productivity due to this cause would be blunted. Thus, in the neighbourhood of full employment, it would be reasonable to expect $f'(v)$ to dominate $D_v'(v,u)$ and moreover both are positive. Therefore $\frac{\partial H}{\partial v} > 0$. (Of course, it is possible that there exists a critical level of the employment ratio such that the two effects cancel each other (and hence the reverse effect valid for values on the 'other side' of the critical level); we ignore this here; elsewhere we have analyzed these other possibilities (cf. Velupillai (1978), (1979(a)).

Thus:

<u>Assumption</u> <u>10</u>:

$$f'(v) > D_v'(v,u)$$

which implies:

$$\frac{\partial\left(\frac{\dot{u}}{u}\right)}{\partial v} = \frac{\partial H}{\partial v} > 0 \qquad (36)$$

The existence of capitalists in a system where the share of wages is unity is of course absurd. Ricardo, in the chapter 'On Profits' summarized this situation quite well:

> The rise in the price of necessaries and in the wages of labour is however limited; for as soon as wages should be equal . . . to . . . the whole receipts of the farmer, there must be an end of accumulation; for no capital can then yield any profit whatever, and no additional labour can be demanded. . . . Long indeed before this period, the very low rate of profits will have arrested

all accumulation . . .

(Ricardo (1951), p. 120)

(cf. also the above quotation from Marx). Thus, however much the employment ratio may rise--even to the extent of full employment--the existence of a capitalist system presupposes the existence of profits. This means there is not only a limit to the level of the share of wages but also a limit to its growth rate--becoming zero or negative after some point. In the spirit, therefore, and within the context in which we made assumptions 7 ~ 10, in particular assumptions 7 and 9, we may be justified in making the following additional assumption:

<u>Assumption</u> <u>11</u>:

$$\left| \frac{\partial}{\partial v}\left[v\ G\ (v,u)\right] + u\frac{\partial H}{\partial u} \right| > \left| H(v,u) \right| \tag{37}$$

This is an eminently reasonable assumption for any capitalistic system. What it intuitively means is what is described above from the writings of Marx and Ricardo; roughly speaking, the proportional growth in the share of wages is dominated by a compounding of the level it has already reached and the growth rate of the employment ratio. We can rewrite (37) as:

$$\left| G(v,u) + v\frac{\partial G}{\partial v} + u\frac{\partial H}{\partial u} \right| > \left| H(v,u) \right| \tag{38}$$

In the form of (38) the inequality is unambiguously reasonable. Finally, let us observe that we have the natural limits $0 < v \leqslant 1$ and $0 < u \leqslant 1$. In addition, we note the following: as the employment ratio falls, there is some value of $v = \bar{v}$ at which, and below which, the share of labour cannot grow; similarly, at some low level $u = \bar{u}$ ('poverty trap'!) of the share of wages, and below it, the employment ratio cannot grow. On the other hand, if the two variables <u>simultaneously</u> take on very low values, then naturally their growth rates would be at least non-negative but more plausibly (certainly for an advanced capitalist economy very low initial conditions would be almost impossible) <u>strictly positive</u>. Lumping these assumptions together, we have:

<u>Assumption 12</u>:

a) $G(0,0) > 0$

b) $H(0,0) > 0$

c) $v = \bar{v}$ such that $G(\bar{v},0) = 0$

d) $u = \bar{u}$ such that $H(0,\bar{u}) = 0$

A final remark before we derive some concrete results about equilibrium: if we take assumptions 7 ~ 12 together with equations and inequalities (28) ~ (36) we have what is sometimes called a generalized SYMBIOTIC model. What we have is a 'partly complementary, partly hostile' relationship between labour and capital.

If now we take (36) and (33) on the one hand and (28) and (31) on the other, we see that the slopes of

$$Q = G^{-1}(0) \tag{39}$$

and

$$R = H^{-1}(0) \tag{40}$$

are positive. A possible configuration of Q and R satisfying assumptions 7 ~ 12 together with (28) ~ (36) would be as in figure 1.

Because of the upper bounds on v and u (being +1) any trajectory that leads to <u>ONE</u> of these bounds must, on economic grounds, lead to a change of 'regime'. These almost unattainable (in a capitalist system) limits are the 'inverses' of Joan Robinson's <u>inflation barrier</u> (cf. Robinson, op. cit.,

p. 48). The ideal solution, from the workers' point of view, would be when Q and R intersect at $u = 1$ and $v = 1$, as shown for a possible Q and R in figure 2. When the functional forms are subject to control--eg., via the parameters in them, or by making one or more of the variables a control variable--then that elusive point on the far northeast corner becomes almost possible, at least in the minds of policy makers.

In the Appendix we prove some theorems on existence, uniqueness, global asymptotic stability and structural stability of the equilibrium.

4. Conclusions and Policy Implications

Theorems 1 and 2 (cf. Appendix) summarize what may be called results of 'Keynesian assumptions' for a capitalist economy. Assuming the conditions that guarantee existence of equilibrium, in this symbiotic system, labour and capital complement each other. High share of wages leading to high consumption; this leads to high quasi-rents for, at least, one class of capitalists who may then demand more goods from other capitalists and thus investment is stimulated leading to high employment. But the system can end up at a low configuration of v and u for an appropriate intersection of Q and R. Thus we can have capitalistic systems of the 'high-wage, high employment' type or 'low wage, low employment' type or even 'high wage, low employment' and 'low wage, high employment' types. So, along with Kaldor, we may also say that the 'heights' and 'slopes' of the Q and R curves, meaning thereby, to a large extent, the 'heights' and 'slopes' of the constituent functions, expresses the particular capitalist society's 'dynamism' and its capitalists' 'animal spirits'. Since the crucial functions that determine the shape of Q and R are:

(a) The profits (or expectation) term in the investment equation;
(b) The technical progress function;
(c) Wage dynamics;
(d) Inflation and its effects on wage dynamics through bargaining;
(e) Capitalists' 'hiring and firing' policies (labour-hoarding effect on the capital-output ratio) and capacity utilization policies;

it is evident that it is possible to achieve the 'high wage, high employment' capitalist economy that many social democratic governments seem to have as their aim--or, as Goodwin put it:

> . . . this golden goose egg theory of capitalism
> (Goodwin (1972), p. 446)

However, it is clear that economic conditions of a very diverse nature can guarantee existence--in particular: Classical, Marxian, Keynesian, etc. Also, the fact that the system is structurally stable means that it is robust for empirical analysis and policy purposes.

It will be evident that the analysis and the model(s) developed owe nothing to neo-classical methodology. Marginal productivities, marginal cost pricing, competitive markets, aggregate production functions, profit maximization and other well known concepts and methodology of neo-classical economics, though not inconsistent with an appropriately modified form of the model developed here, are <u>not necessary for any of the conclusions</u>. Marginal analysis, the 'bread-and-butter' of neo-classical economics, is a natural corollary of the choice--theoretic or optimization framework of microeconomics. Thus recent research in so-called 'microeconomic foundations of macroeconomics' or the attempts to make compatible Walrasian microeconomics and Keynesian macroeconomics, ultimately reduce to providing choice-theoretic frameworks such that the two fields can be reconciled. The disequilibrium macrodynamics in this tradition, inspired by Barro-Grossman (1976), Malinvaud (1977, 1980), etc., are, therefore, ultimately 'marginal' in substance. However, the passage from the micro choice-theoretic framework to macro seems to be as ad-hoc as the Keynesian macroeconomics that has been criticized. (Appeal is made to the 'representative firm', 'the representative consumer', etc. Analytically, such assumptions are identical to the assumption of homogeneous labour and equal organic composition of capital on a tradition richer in dynamics.) To say, therefore, that the model developed in this paper is not inconsistent with neoclassical methodology only means that the constituent functional relations can be given a microeconomic choice theoretic rationale. That this has not been attempted here is not an accident--nor is it to condone ad-hoc macroeconomics. The fundamental dynamics of a capitalist society must be such as to reproduce the relations of exchange, production and distribution that characterize it. It is our conjecture that there are macroeconomic relations that cannot (and indeed should not) be given microfoundations. Borrowing Pasinetti's celebrated statement used in a different--but related--context, we may say that these relations are macroeconomic because they cannot be otherwise (cf. Pasinetti (1974), p. 118). The foundations for such relations should be sought not in microeconomics as conventionally understood but in macro-politics, political economy and, more generally, in those forces that determine the class structure of society (cf. also Hicks in Harcourt, ed. (1977), pp. 373-374 and Hicks (1979), pp. vii-viii). It is perhaps ironical that such a conjecture can even be extrapolated from the recent results of Sonnenschein (1972, 1973a, 1973b), and Debreu (1974).

As a framework for macropolicy discussions, however, there are serious limitations in the structure of the model presented in this paper. In particular the disequilibrium is confined to the labour market. The Keynesian problem of effective demand is, therefore, only a shadow. It is, nevertheless, not difficult to introduce a simultaneous disequilibrium also in the goods market. The analysis loses the simplicity of two dimensions and this is the main reason for avoiding this important question explicitly. In Fitoussi-Velupillai (1981) this deficiency is rectified, and we consider the dynamics of functional income distribution in the presence of disequilibria in both the labour and goods market.

Beyond that the further deficiencies are, in particular, three:

(a) Assumption of a <u>closed</u> economy.

(b) <u>No</u> explicit role for a government.

(c) It is a real and <u>not</u> <u>a</u> <u>monetary</u> economy.

The obvious extensions of the model above, <u>if</u> <u>found</u> <u>useful</u>, would be to relax these assumptions. We do not subscribe to the view that disaggregation is essential for the questions we have chosen to discuss--employment and functional income distribution. This space, the space of v-u, appears to be the natural space for the Cambridge economists in particular and Post Keynesians in general--as indeed for the great Classical economists and Marx. It is, after all, the space of 'magnificent dynamics'.

A P P E N D I X

We will now prove some theorems on existence, uniqueness, global asymptotic stability and structural stability of the equilibrium. For convenience in the proofs we slightly distort the order and prove, first, uniqueness by assuming existence and then prove existence and stability properties after that.

Theorem 1: (Uniqueness)

Assume that Q and R intersect in $0 < v \leq 1$ and $0 < u \leq 1$ such that their tangent lines are distinct. Under assumptions 7 ~ 10 and 12 for the dynamical system in v and u given by (18) and (27) with (36), (33), (28) and (31), the equilibrium is unique.

(For the assumptions of theorem 1, a possible configuration of the dynamics in the phase-plane of v and u is given in figure 3.)

Proof:

Clearly, there are no closed orbits. This is because all trajectories in each basic region (cf. Hirsch-Smale (1974), p. 267) are monotone. By assumption there is an equilibrium in $0 < v \leq 1$, $0 < u \leq 1$. By (28), (31) and (33), (36) the slopes of Q and R are positive and monotone. Hence the equilibrium is unique.

Now, the following is a theorem giving sufficient conditions for existence of equilibrium.

Theorem 2: (Existence - uniqueness)

If the direct effect of increase in employment on proportional growth in employment is greater than the indirect effects due to increases in the share of wages (i.e., if $\left|\frac{\partial G}{\partial v}\right| > \left|\frac{\partial G}{\partial u}\right|$) and if similar condition holds for the share of wages (i.e., if $\left|\frac{\partial H}{\partial u}\right| > \left|\frac{\partial H}{\partial v}\right|$), then there exists an equilibrium which is unique.

Proof:

Only existence in the first part of the theorem needs proof because uniqueness follows from the proof of theorem 1. As in theorem 1, closed orbits are ruled out because every trajectory in each basic region is monotone. Hence, from the fact that the limit sets of trajectories are closed and invariant and from the fact that every basic region together with its closure is either positively or negatively invariant it follows, by the Poincaré-Bendixson theorem that all w-limit points are equilibria. (For an explanation of the technical terms used in this proof see any good book on the Qualitative Theory of Differential Equations such as Coddington and Levison (1955), Nemytskii-Stepanov (1964) or Hirsch-Smale (1974).

Q.E.D.

It must be remarked that the word 'equilibrium' carries with it no normative significance. The most plausible equilibrium for an economy with our assumptions would be well inside the unit square determined by $0 \leq v \leq 1$ and $0 \leq u \leq 1$ --though perhaps in the upper northeast corner rather than the lower southwest part of the square. All this simply means that equilibrium, in our system, will be characterized by under-full employment and a value for the share of wages well under unity.

Next, we prove two stability theorems.

Theorem 3: (Global Asymptotic Stability)

Under assumption 7 ~ 12 for the dynamical system in v and u given by (18) and (27) with (36), (33), (28) and (31) and the conditions for existence assumed in theorem 2, the equilibrium is globally asymptotically stable.

Proof:

We use the sufficiency conditions of Olech (1963). By Olech's theorem (op. cit.) sufficient conditions for global asymptotic stability are (in the phase-plane):

(a) Negative trace of the Jacobian of a Dynamical System

(b) Positive determinant from the Jacobian

(c) Either the product of the diagonal terms or the product of the off-diagonal terms must be non-zero

(a), (b), (c) to hold, in general, on the plane. However, for our purposes, any initial conditions must be in the unit square determined by $0 \leqslant v \leqslant 1$ and $0 \leqslant u \leqslant 1$; therefore we are only interested in the non-negative orthant circumscribed by the natural economic limits on v and u.

Now, from assumption 11 and equations (29) and (32) we see that condition a) of Olech's theorem is satisfied.

From the conditions of the first part of theorem 2 (guaranteeing existence) and from (36), (33), (28) and (31) we see that the determinant from the Jacobian is positive.

Finally, condition c) is immediate from any combination of the four inequalities (36), (33), (28) and (31) (and the non-negativity of v and u).

Thus, the equilibrium, when feasible, is globally asymptotically stable. (Even otherwise the theorem is valid but irrelevant from an economic point of view.)

Theorem 4: (Structural Stability)

The dynamic system for v and u given by (18) and (27) under (36), (33), (28) and (31) and assumptions 7 ~ 10 and 12 is structurally stable.

Proof:

Since we are only interested in the structural stability of a feasible equilibrium it is immediate that condition c of theorem 2 in Hirsch-Smale (op. cit.), p. 314 is satisfied.

Since closed orbits are ruled out by the above theorems condition b) of the same theorem in Hirsch-Smale is also satisfied.

Finally, direct computation verifies that condition (a) of the same theorem in Hirsch-Smale is also satisfied.

Thus, the dynamical system is structurally stable.

Q.E.D.

Some remarks about the structural stability theorem may not be out of place. Firstly we may point out that elsewhere (cf. Velupillai (1979c)) we have gone into the technical details of the definition of structural stability and the link between the Hirsch-Smale definition and the 'classical' (Andronov et al. (op. cit.), De Baggis (1952)) definition. Secondly, we feel that for an operational model, i.e., where observations are important, structural stability is much more important than ordinary stability. Roughly speaking, structural stability means that the qualitative behaviour of the system is invariant to small perturbations in the parameters. This was why Andronov and Pontryagin (1937) called systems that are structurally stable as 'Coarse Systems'.

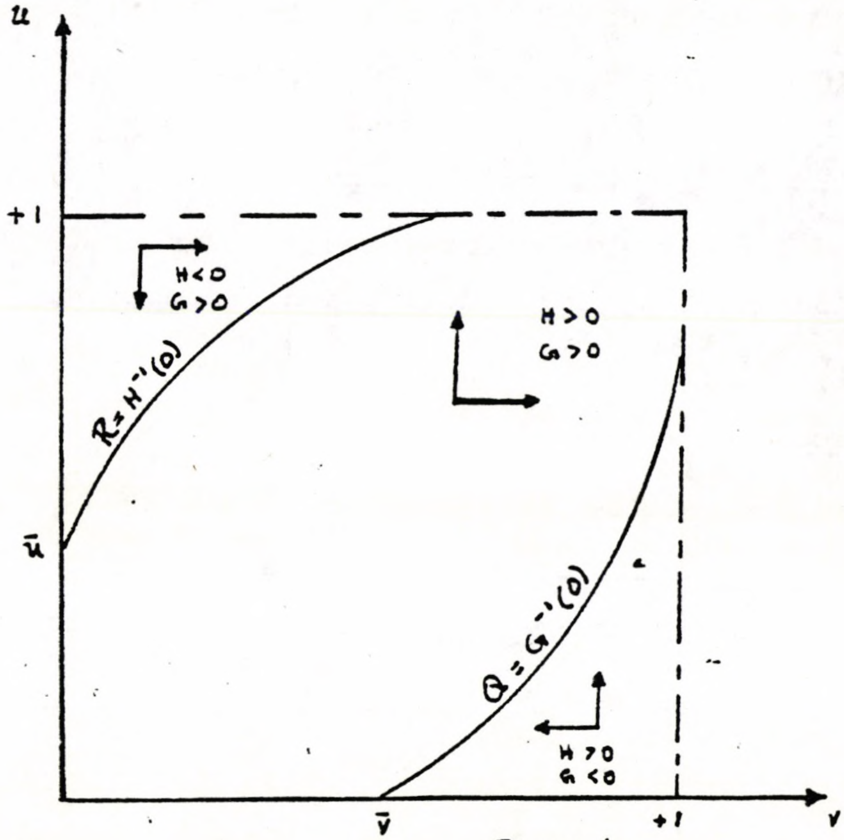

Figure 1

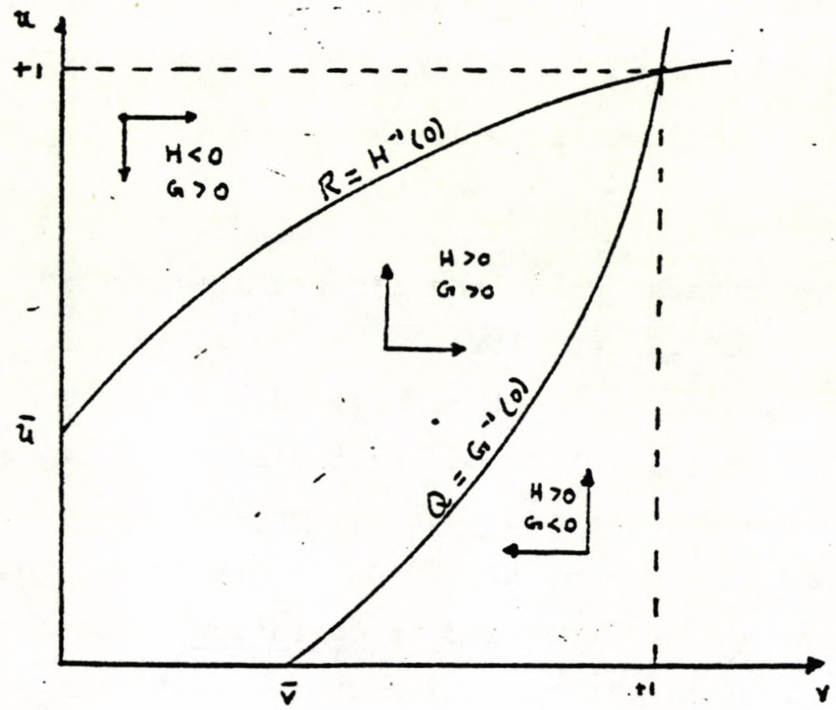

Figure 2

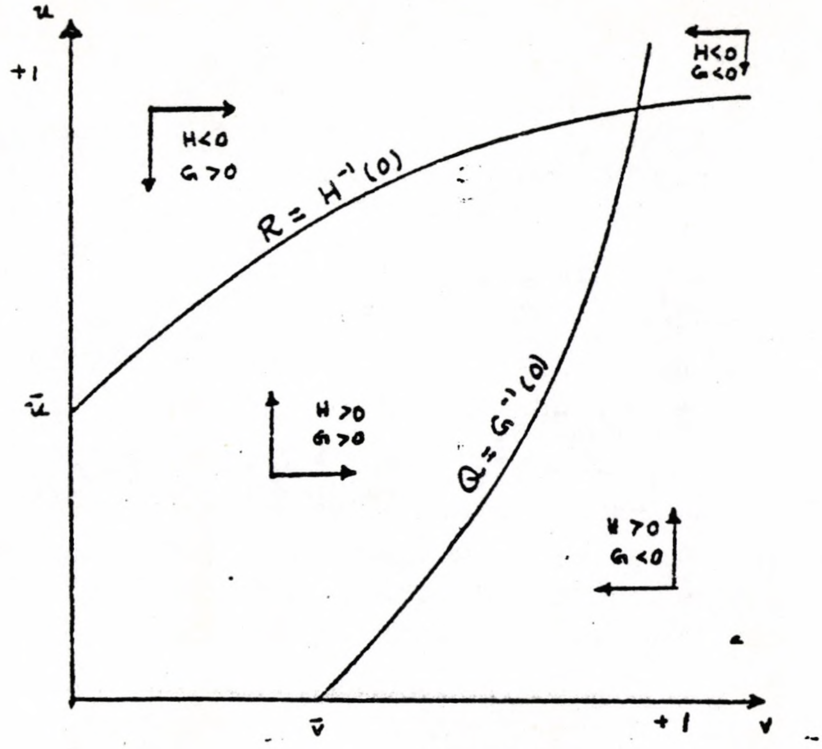

Figure 3.

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